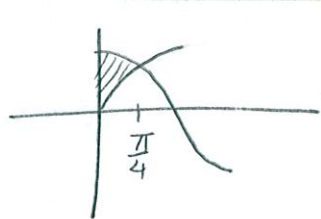


Find the y-coordinate of the center of mass of the region in the first quadrant bounded by $y = \cos x$, $y = \sin x$ and $x = 0$. Your final answer must be a number, not an integral. SCORE: ____ / 25 PTS



$$\int_0^{\pi/4} (\cos x - \sin x) dx = (\sin x + \cos x) \Big|_0^{\pi/4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 = \sqrt{2} - 1$$

$$\frac{1}{2} \int_0^{\pi/4} (\cos^2 x - \sin^2 x) dx = \frac{1}{2} \int_0^{\pi/4} \cos 2x dx$$

(4)

$$= \frac{1}{4} \sin 2x \Big|_0^{\pi/4}$$

$$= \frac{1}{4} (1) = \frac{1}{4}$$

$$\bar{y} = \frac{1}{\sqrt{2} - 1} \cdot \frac{1}{4}$$

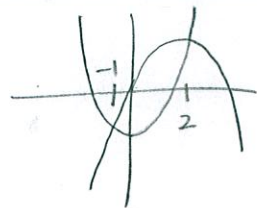
$$= \frac{\sqrt{2} + 1}{4}$$

(3) EXCEPT AS NOTED

Find the area of the region between $y = 4x^2 - 12$ and $y = 6x - 2x^2$ for $-2 \leq x \leq 4$.

SCORE: ____ / 25 PTS

Your final answer must be a number, not an integral.



$$4x^2 - 12 = 6x - 2x^2$$

$$6x^2 - 6x - 12 = 0$$

$$6(x^2 - x - 2) = 0$$

$$6(x + 1)(x - 2) = 0$$

$$x = -1, 2$$

$$\int_{-2}^{-1} (4x^2 - 12 - (6x - 2x^2)) dx + \int_{-1}^2 (6x - 2x^2 - (4x^2 - 12)) dx$$

$$+ \int_2^4 (4x^2 - 12 - (6x - 2x^2)) dx$$

$$= \int_{-2}^{-1} (6x^2 - 6x - 12) dx + \int_{-1}^2 (-6x^2 + 6x + 12) dx + \int_2^4 (6x^2 - 6x - 12) dx$$

$$= (2x^3 - 3x^2 - 12x) \Big|_{-2}^{-1} + (-2x^3 + 3x^2 + 12x) \Big|_{-1}^2 + (2x^3 - 3x^2 - 12x) \Big|_2^4$$

$$= 2(-1 - 8) - 3(1 - 4) - 12(-1 - 2) - 2(8 - 1) + 3(4 - 1) + 12(2 - 1)$$

$$+ 2(64 - 8) - 3(16 - 4) - 12(4 - 2) = 14 + 9 - 12 - 18 + 9 + 36 + 112 - 36 - 24$$

$$= 190$$

4

6

6

6

Find the length of the curve $y = \int_{-1}^x \sqrt{e^{4t}(e^{4t} + 2)} dt$ for $0 \leq x \leq 2$.

SCORE: ____ / 20 PTS

Your final answer must be a number, not an integral.

$$\frac{dy}{dx} = \sqrt{e^{4x}(e^{4x} + 2)} \text{ BY FTC PART 1}$$

$$\int_0^2 \sqrt{1 + \left(\sqrt{e^{4x}(e^{4x} + 2)} \right)^2} dx$$

$$= \int_0^2 \sqrt{e^{8x} + 2e^{4x} + 1} dx$$

$$= \int_0^2 (e^{4x} + 1) dx$$

$$= \left(\frac{1}{4} e^{4x} + x \right) \Big|_0^2 = \frac{1}{4}(e^8 - 1) + 2 = \frac{1}{4}e^8 + \frac{7}{4}$$

(4½) EACH EXCEPT AS NOTED

(2)

At the start of each quarter, there is a 12 day window in which students may drop a course without a record of their enrolment in the class appearing on their transcript. Suppose a student is randomly selected among those who dropped a class during this quarter's 12 day window. Let X be the number of days after the start of the quarter that the student dropped their class. Suppose that the probability density function for X is given by

SCORE: ____ / 30 PTS

$$f(x) = \begin{cases} k \sin \frac{\pi x}{24}, & x \in [0, 12] \\ 0, & x \notin [0, 12] \end{cases}$$

[a] Find the probability that a student dropped a class during the second half of the window.

$$\begin{aligned} \textcircled{4} \quad \int_0^{12} k \sin \frac{\pi x}{24} dx &= 1 \\ k \left[-\frac{24}{\pi} \cos \frac{\pi x}{24} \right]_0^{12} &= 1 \\ \textcircled{4} \quad -\frac{24k}{\pi} (\cos \frac{\pi}{2} - 1) &= 1 \\ \textcircled{2} \quad k &= \frac{\pi}{24} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \frac{\pi}{24} \int_6^{12} \sin \frac{\pi x}{24} dx &= \frac{\pi}{24} \left[-\frac{24}{\pi} \cos \frac{\pi x}{24} \right]_6^{12} \\ &= -(\cos \frac{\pi}{2} - \cos \frac{\pi}{4}) \\ \textcircled{5} \quad &= \frac{\sqrt{2}}{2} \approx 0.7071 \text{ or } 71\% \end{aligned}$$

[b] Find the median number of days before students drop their classes.

$$\begin{aligned} \frac{\pi}{24} \int_0^m \sin \frac{\pi x}{24} dx &= \frac{1}{2} \quad \textcircled{4} \\ \frac{\pi}{24} \left[-\frac{24}{\pi} \cos \frac{\pi x}{24} \right]_0^m &= \frac{1}{2} \\ \cos \frac{\pi m}{24} - 1 &= -\frac{1}{2} \quad \textcircled{4} \\ \cos \frac{\pi m}{24} &= \frac{1}{2} \end{aligned}$$

$\frac{\pi m}{24} = \frac{\pi}{3}$
 $m = 8 \text{ DAYS}$
 $\textcircled{2}$

A solid of revolution has volume $\int_1^3 2\pi(y^2 - \ln y)(y+4) dy$. Find the equation of the axis of revolution,

SCORE: ____ / 15 PTS

and the equations of the boundaries of the region being revolved. Sketch the region being revolved.

Do NOT use the x - nor y -axes as boundaries nor the axis of revolution.

Equation of axis of revolution:

$y = -4$ (2)

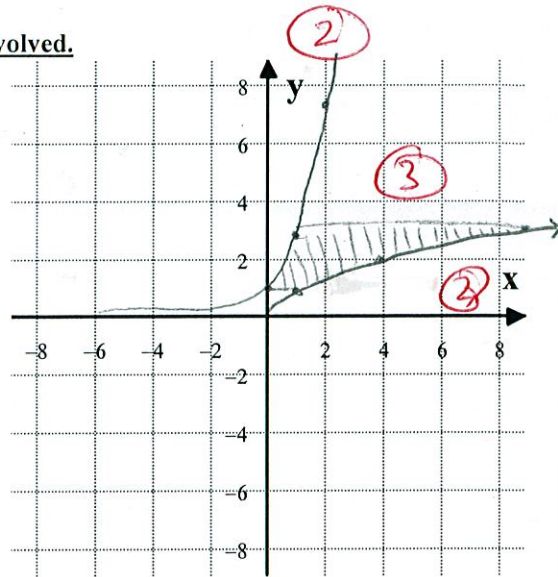
Equations of boundaries:

$y = 1$ (1)

$y = 3$ (1)

$x = y^2$ (2)

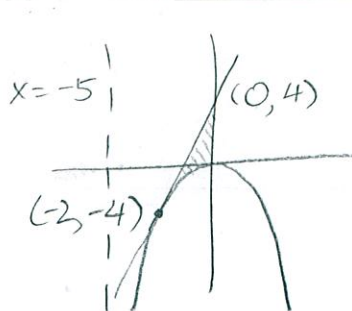
$x = \ln y$ or $y = e^x$ (2)



The region bounded by $y = -x^2$, $y = 4x + 4$ and $x = 0$ is revolved around the line $x = -5$.

SCORE: ____ / 35 PTS

- [a] Write, **BUT DO NOT EVALUATE**, a single integral for the volume of the resulting solid.



$$\begin{aligned} -x^2 &= 4x + 4 \\ 0 &= x^2 + 4x + 4 \\ 0 &= (x + 2)^2 \\ x &= -2 \end{aligned}$$

$$\int_{-2}^0 2\pi (x+5) (4x+4 - (-x^2)) dx$$

(4) (3) (4) (4)

- [b] If you used the disk or washer method in [a], write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the shell method.
If you used the shell method in [a], write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the disk or washer method.

$$\begin{aligned} &\int_{-4}^0 \pi \left[\left(-\sqrt{y} - 5 \right)^2 - \left(\frac{y-4}{4} - 5 \right)^2 \right] dy \\ &+ \int_0^4 \pi \left[\left(0 - 5 \right)^2 - \left(\frac{y-4}{4} - 5 \right)^2 \right] dy \end{aligned}$$

(3) (4) (3) (2) (5)

$$\begin{aligned} y = -x^2 &\rightarrow x = \pm\sqrt{-y} \\ y = 4x + 4 &\rightarrow x = \frac{y-4}{4} \end{aligned}$$